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Note

Multiple scattering by particles embedded in an absorbing medium. 2. Radiative transfer equation

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ABSTRACT

This paper continues a systematic theoretical analysis of electromagnetic scattering by a group of arbitrarily sized, shaped, and oriented particles embedded in an absorbing, homogeneous, isotropic, and unbounded medium. The previously developed microphysical approach is used to derive the generalized form of the radiative transfer equation (RTE) applicable to a large group of sparsely, randomly, and uniformly distributed particles. The derivation of the RTE directly from the macroscopic Maxwell equations yields unambiguous and definitive analytical expressions for the participating quantities and thereby fully resolves the lasting controversy caused by the conflicting outcomes of several phenomenological approaches.

1. Introduction

The specific form of the radiative transfer equation (RTE) and the physical meaning of the participating quantities in the important case of an absorbing host medium have been the subject of a lasting controversy (see e.g., [1–17] and references therein). This controversy can largely be attributed to the conflicting outcomes of several phenomenological studies based on the common assumption that the RTE exists even when the host medium is absorbing and has the standard mathematical structure [18,19] but needs modified participating quantities.

As always, the best and most straightforward way to resolve such a controversy is to (i) adhere only to quantities that can be measured directly [2,10,16] and (ii) use a microphysical rather than a phenomenological approach and derive rather than guess the final equations [14,17]. In this particular case, the microphysical approach should be based directly on the macroscopic Maxwell equations in much the same way as it has been done for the case of a non-absorbing host medium [20–22].

This paper is the third part of a series. The first paper presented a general and systematic analysis of the problem of (single) electromagnetic scattering by an arbitrary finite fixed object embedded in an absorbing, homogeneous, isotropic, and unbounded medium [16]. We used the volume integral equation to derive generalized formulas of the far-field approximation and introduced direct optical observables such as the phase and extinction matrices. The second paper was concerned with multiple scattering by a finite group of particles [17]. We used the volume integral equation to derive generalized Foldy–Lax equations and their order-of-scattering form. The far-field version of the Foldy–Lax equations was used to derive the transport equation for the coherent field generated by a large group of sparsely, randomly, and uniformly distributed particles. In this third paper, we complete the derivation of the full RTE.

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The first two papers [16,17] illustrate that the generalization of the existing single- and multiple-scattering theories to the case of an absorbing host medium is rather straightforward in almost all respects and, in essence, requires a careful step-by-step repetition of the analytical derivations presented in exquisite detail in the monographs [21,23]. Perhaps the only important subtlety to keep in mind is that the method of stationary phase and the Saxon expansion of a plane wave in spherical waves are applicable only to expressions involving purely complex exponentials of the type $\exp(ik\alpha)$ with a real-valued $k\alpha$. After reading [16,17] one should have a rather clear idea of what changes in the main analytical results should be expected upon letting the host medium be absorbing. Therefore, in this paper, we will save space by skipping all intermediate analytical derivations detailed in Sections 8.5–8.10 of [21] and will focus on the qualitative explanation of the final result and the discussion of its physical meaning and implications.

2. Radiative transfer equation

Consider the scattering of a plane electromagnetic wave by a large group of N particles randomly distributed throughout a large but finite scattering volume V (Fig. 1). The host medium can be absorbing, but otherwise it is assumed to be infinite, homogeneous, linear, and isotropic. The particles are assumed to have the same constant permeability, but may have different and spatially varying permittivities. These assumptions allow us to use the results of [16,17].

In accordance with [20,21], the microphysical derivation of the RTE involves several basic steps. The first one is to assume that each particle is located in the far-field zones of all the other particles and that the observation point is also located in the far-field zones of all the particles filling the scattering volume. This assumption leads to a drastic simplification of the Foldy–Lax equations (FLEs) wherein they are converted from a system of volume integral equations into a system of linear algebraic equations [17].

The order-of-scattering expansion of the far-field FLEs, Eq. (28) of [17], allows one to represent the total electric field at a point in space as a sum of contributions arising from all possible particle sequences. The second step is to assume the validity of the Twersky approximation [17,24] according to which all sequences going through a particle more than once can be neglected. This is justified provided that N is very large.

The third step is to assume full ergodicity of the random N-particle group, which allows one to replace averaging over time by averaging over particle positions and states [21,22].

The fourth step is to assume that the position and state of each particle are statistically independent of each other and of those of all the other particles, and that the spatial distribution of the particles throughout the scattering volume is random and statistically uniform.

The fifth step is to characterize the multiply scattered radiation by the coherency dyadic $\overleftrightarrow{C}(\mathbf{r}) = \langle \mathbf{E}(\mathbf{r},t) \otimes \mathbf{E}^*(\mathbf{r},t)\rangle_t \approx \langle \mathbf{E}(\mathbf{r}) \otimes \mathbf{E}^*(\mathbf{r})\rangle_{\mathbf{R},\xi}$, where $\mathbf{E}$ is the electric field vector, t is time, $\mathbf{r}$ is the position vector of the observation point, the asterisk denotes complex conjugation, $\otimes$ is the dyadic product sign, the subscript t denotes averaging over time, and the subscripts $\mathbf{R}$ and ξ denote averaging over all particle coordinates and states, respectively. Because of the averaging over particle

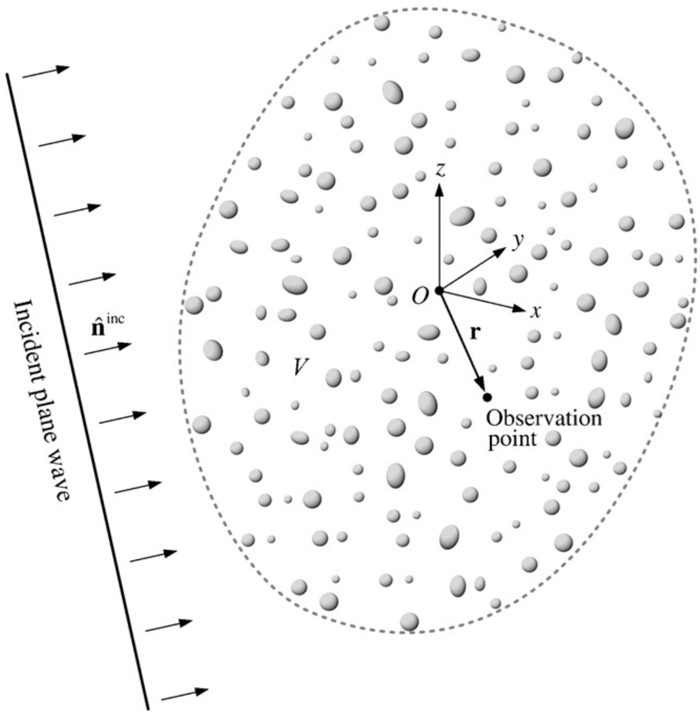

Fig. 1. Electromagnetic scattering by a large group of particles sparsely distributed throughout a macroscopic volume V.

coordinates, $\overleftrightarrow{C}(\mathbf{r})$ is a continuous function of the position vector. Furthermore, it can be used to define derivative quantities that are observable directly.

The next major assumption in the derivation of the RTE is that all diagrams with crossing connectors in the Twersky expansion of the coherency dyadic can be neglected [20,21]. Careful analytical evaluation of the cumulative position- and state-averaged contribution of all diagrams with vertical connectors coupled with the assumption that N is very large leads to the ladder approximation for the coherency dyadic [20,21]. The expanded expression for the ladder coherency dyadic has the form of an angular decomposition in terms of the so-called ladder-specific coherency dyadic $\overleftrightarrow{\Sigma}_{\mathrm{L}}(\mathbf{r},\hat{\mathbf{q}})$:

$$\overleftrightarrow{C}(\mathbf{r}) \approx \overleftrightarrow{C}_{\mathrm{L}}(\mathbf{r}) = \int_{4\pi} \mathrm{d}\hat{\mathbf{q}}\, \overleftrightarrow{\Sigma}_{\mathrm{L}}(\mathbf{r},\hat{\mathbf{q}}), \tag{1}$$

where the integration is performed over all propagation directions as specified by the unit vector $\hat{\mathbf{q}}$. Furthermore, it is straightforward to show that the specific coherency dyadic satisfies an integral RTE. The ladder-specific coherency dyadic can, in turn, be used to define the so-called specific intensity column vector,

$$\tilde{\mathbf{I}}(\mathbf{r},\hat{\mathbf{q}}) = \begin{bmatrix} \tilde{I}(\mathbf{r},\hat{\mathbf{q}}) \\ \tilde{Q}(\mathbf{r},\hat{\mathbf{q}}) \\ \tilde{U}(\mathbf{r},\hat{\mathbf{q}}) \\ \tilde{V}(\mathbf{r},\hat{\mathbf{q}}) \end{bmatrix} = \mathrm{Re}\left(\frac{k_1}{2\omega\mu_1}\right) \begin{bmatrix} \hat{\boldsymbol{\theta}}(\hat{\mathbf{q}}) \cdot \overleftrightarrow{\Sigma}_{\mathrm{L}}(\mathbf{r},\hat{\mathbf{q}}) \cdot \hat{\boldsymbol{\theta}}(\hat{\mathbf{q}}) + \hat{\boldsymbol{\phi}}(\hat{\mathbf{q}}) \cdot \overleftrightarrow{\Sigma}_{\mathrm{L}}(\mathbf{r},\hat{\mathbf{q}}) \cdot \hat{\boldsymbol{\phi}}(\hat{\mathbf{q}}) \\ \hat{\boldsymbol{\theta}}(\hat{\mathbf{q}}) \cdot \overleftrightarrow{\Sigma}_{\mathrm{L}}(\mathbf{r},\hat{\mathbf{q}}) \cdot \hat{\boldsymbol{\theta}}(\hat{\mathbf{q}}) - \hat{\boldsymbol{\phi}}(\hat{\mathbf{q}}) \cdot \overleftrightarrow{\Sigma}_{\mathrm{L}}(\mathbf{r},\hat{\mathbf{q}}) \cdot \hat{\boldsymbol{\phi}}(\hat{\mathbf{q}}) \\ -\hat{\boldsymbol{\theta}}(\hat{\mathbf{q}}) \cdot \overleftrightarrow{\Sigma}_{\mathrm{L}}(\mathbf{r},\hat{\mathbf{q}}) \cdot \hat{\boldsymbol{\phi}}(\hat{\mathbf{q}}) - \hat{\boldsymbol{\phi}}(\hat{\mathbf{q}}) \cdot \overleftrightarrow{\Sigma}_{\mathrm{L}}(\mathbf{r},\hat{\mathbf{q}}) \cdot \hat{\boldsymbol{\theta}}(\hat{\mathbf{q}}) \\ \mathrm{i}[\hat{\boldsymbol{\phi}}(\hat{\mathbf{q}}) \cdot \overleftrightarrow{\Sigma}_{\mathrm{L}}(\mathbf{r},\hat{\mathbf{q}}) \cdot \hat{\boldsymbol{\theta}}(\hat{\mathbf{q}}) - \hat{\boldsymbol{\theta}}(\hat{\mathbf{q}}) \cdot \overleftrightarrow{\Sigma}_{\mathrm{L}}(\mathbf{r},\hat{\mathbf{q}}) \cdot \hat{\boldsymbol{\phi}}(\hat{\mathbf{q}})] \end{bmatrix}, \tag{2}$$

which also satisfies an integral RTE. In the above formula, $k_1 = k_1' + \mathrm{i}k_1''$ is the complex-valued wave number of the host medium, ω is the angular frequency, μ_1 is the magnetic permeability of the host medium, and $\hat{\boldsymbol{\theta}}(\hat{\mathbf{q}})$ and $\hat{\boldsymbol{\phi}}(\hat{\mathbf{q}})$ are the unit vectors in the local spherical coordinate system corresponding to the propagation direction $\hat{\mathbf{q}}$. Finally, the integral RTE for $\tilde{\mathbf{I}}(\mathbf{r},\hat{\mathbf{q}})$ can be converted into the familiar integro-differential form

$$\hat{\mathbf{q}} \cdot \nabla \tilde{\mathbf{I}}(\mathbf{r},\hat{\mathbf{q}}) = -2k''_1 \tilde{\mathbf{I}}(\mathbf{r},\hat{\mathbf{q}}) - n_0 \langle \mathbf{K}(\hat{\mathbf{q}}) \rangle_\xi \tilde{\mathbf{I}}(\mathbf{r},\hat{\mathbf{q}}) + n_0 \int_{4\pi} \mathrm{d}\hat{\mathbf{q}}' \langle \mathbf{Z}(\hat{\mathbf{q}},\hat{\mathbf{q}}') \rangle_\xi \tilde{\mathbf{I}}(\mathbf{r},\hat{\mathbf{q}}'), \tag{3}$$

where $\langle \mathbf{K}(\hat{\mathbf{q}}) \rangle_\xi$ and $\langle \mathbf{Z}(\hat{\mathbf{q}},\hat{\mathbf{q}}') \rangle_\xi$ are the extinction and the phase matrix, respectively, averaged over all particle states and $n_0 = N/V$ is the particle number density.

The specific intensity column vector can be decomposed into the coherent and diffuse parts,

$$\tilde{\mathbf{I}}(\mathbf{r},\hat{\mathbf{q}}) = \delta(\hat{\mathbf{q}} - \hat{\mathbf{n}}^{\mathrm{inc}}) \mathbf{I}_{\mathrm{c}}(\mathbf{r}) + \tilde{\mathbf{I}}_{\mathrm{d}}(\mathbf{r},\hat{\mathbf{q}}), \tag{4}$$

each satisfying its own RTE:

$$\hat{\mathbf{n}}^{\mathrm{inc}} \cdot \nabla \mathbf{I}_{\mathrm{c}}(\mathbf{r}) = -2k''_1 \mathbf{I}_{\mathrm{c}}(\mathbf{r}) - n_0 \langle \mathbf{K}(\hat{\mathbf{n}}^{\mathrm{inc}}) \rangle_\xi \mathbf{I}_{\mathrm{c}}(\mathbf{r}), \tag{5}$$

$$\hat{\mathbf{q}} \cdot \nabla \tilde{\mathbf{I}}_{\mathrm{d}}(\mathbf{r},\hat{\mathbf{q}}) = -2k''_1 \tilde{\mathbf{I}}_{\mathrm{d}}(\mathbf{r},\hat{\mathbf{q}}) - n_0 \langle \mathbf{K}(\hat{\mathbf{q}}) \rangle_\xi \tilde{\mathbf{I}}_{\mathrm{d}}(\mathbf{r},\hat{\mathbf{q}}) + n_0 \int_{4\pi} \mathrm{d}\hat{\mathbf{q}}' \langle \mathbf{Z}(\hat{\mathbf{q}},\hat{\mathbf{q}}') \rangle_\xi \tilde{\mathbf{I}}_{\mathrm{d}}(\mathbf{r},\hat{\mathbf{q}}') + n_0 \langle \mathbf{Z}(\hat{\mathbf{q}},\hat{\mathbf{n}}^{\mathrm{inc}}) \rangle_\xi \mathbf{I}_{\mathrm{c}}(\mathbf{r}), \tag{6}$$

where $\hat{\mathbf{n}}^{\mathrm{inc}}$ is the direction of incidence of the external plane electromagnetic wave (Fig. 1). $\mathbf{I}_{\mathrm{c}}$ reduces to the Stokes column vector of the incident wave at the illuminated boundary of the scattering volume, but is subject to exponential attenuation and, possibly, the effect of dichroism inside the volume. The exponential attenuation is caused by both the host medium and the particles.

3. Discussion

Our use of the method of stationary phase and the Saxon expansion of a plane wave in spherical waves in the derivation of the RTE is implicitly based on the assumption that $k_1'' \ll k_1'$. This assumption is quite reasonable because otherwise absorption by the macroscopic host medium would extinguish any observable consequence of multiple scattering.

The only formal difference of Eqs. (3), (5), and (6) from the corresponding equations in the case of a non-absorbing host medium [20,21] is the presence of terms proportional to k_1''. These terms describe the absorption of electromagnetic energy by the host medium and vanish if the refractive index of the host medium is real valued. Furthermore, Eqs. (3), (5), and (6) can be made mathematically equivalent to those in [20,21] by the introduction of a new "effective extinction matrix" according to

$$\mathbf{K}^{\mathrm{eff}}(\hat{\mathbf{q}}) = (2k''_1/n_0)\ \mathrm{diag}[1,1,1,1] + \langle \mathbf{K}(\hat{\mathbf{q}}) \rangle_\xi. \tag{7}$$

Indeed, we then have

$$\hat{\mathbf{q}} \cdot \nabla \tilde{\mathbf{I}}(\mathbf{r},\hat{\mathbf{q}}) = -n_0 \mathbf{K}^{\mathrm{eff}}(\hat{\mathbf{q}}) \tilde{\mathbf{I}}(\mathbf{r},\hat{\mathbf{q}}) + n_0 \int_{4\pi} \mathrm{d}\hat{\mathbf{q}}' \langle \mathbf{Z}(\hat{\mathbf{q}},\hat{\mathbf{q}}') \rangle_\xi \tilde{\mathbf{I}}(\mathbf{r},\hat{\mathbf{q}}'), \tag{8}$$

$$\hat{\mathbf{n}}^{\mathrm{inc}} \cdot \nabla \mathbf{I}_{\mathrm{c}}(\mathbf{r}) = -n_0 \mathbf{K}^{\mathrm{eff}}(\hat{\mathbf{n}}^{\mathrm{inc}}) \mathbf{I}_{\mathrm{c}}(\mathbf{r}), \tag{9}$$

$$\hat{\mathbf{q}} \cdot \nabla \tilde{\mathbf{I}}_{\mathrm{d}}(\mathbf{r}, \hat{\mathbf{q}}) = -n_0 \mathbf{K}^{\mathrm{eff}}(\hat{\mathbf{q}}) \tilde{\mathbf{I}}_{\mathrm{d}}(\mathbf{r}, \hat{\mathbf{q}}) + n_0 \int_{4\pi} \mathrm{d}\hat{\mathbf{q}}' \langle \mathbf{Z}(\hat{\mathbf{q}}, \hat{\mathbf{q}}') \rangle_\xi \tilde{\mathbf{I}}_{\mathrm{d}}(\mathbf{r}, \hat{\mathbf{q}}') + n_0 \langle \mathbf{Z}(\hat{\mathbf{q}}, \hat{\mathbf{n}}^{\mathrm{inc}}) \rangle_\xi \mathbf{I}_{\mathrm{c}}(\mathbf{r}). \tag{10}$$

This formal mathematical equivalence ensures the direct applicability of the solution approaches described in Chapter 10 of [21].

The RTE (3) becomes considerably simpler in the case of a macroscopically isotropic and mirror-symmetric scattering medium:

$$\hat{\mathbf{q}} \cdot \nabla \tilde{\mathbf{I}}(\mathbf{r}, \hat{\mathbf{q}}) = -[2k''_1 + n_0 \langle C_{\mathrm{ext}} \rangle_\xi] \tilde{\mathbf{I}}(\mathbf{r}, \hat{\mathbf{q}}) + n_0 \int_{4\pi} \mathrm{d}\hat{\mathbf{q}}' \langle \mathbf{Z}(\hat{\mathbf{q}}, \hat{\mathbf{q}}') \rangle_\xi \tilde{\mathbf{I}}(\mathbf{r}, \hat{\mathbf{q}}'), \tag{11}$$

where $\langle C_{\mathrm{ext}} \rangle_\xi = \langle K_{11} \rangle_\xi$ is the average extinction cross-section per particle. If the medium is plane-parallel then

$$u \frac{\mathrm{d}\tilde{\mathbf{I}}(\tau, \hat{\mathbf{q}})}{\mathrm{d}\tau} = -\tilde{\mathbf{I}}(\tau, \hat{\mathbf{q}}) + \frac{1}{C_{\mathrm{ext}}^{\mathrm{eff}}} \int_{4\pi} \mathrm{d}\hat{\mathbf{q}}' \langle \mathbf{Z}(\hat{\mathbf{q}}, \hat{\mathbf{q}}') \rangle_\xi \tilde{\mathbf{I}}(\tau, \hat{\mathbf{q}}'), \tag{12}$$

where $\mathrm{d}\tau = n_0 C_{\mathrm{ext}}^{\mathrm{eff}} \mathrm{d}z$ is the differential element of the optical depth,

$$C_{\mathrm{ext}}^{\mathrm{eff}} = 2k''_1 / n_0 + \langle C_{\mathrm{ext}} \rangle_\xi \tag{13}$$

is the "effective extinction cross section", $u = -\cos\theta$ is the direction cosine, and θ is the zenith angle of the propagation direction $\hat{\mathbf{q}}$. The z-axis of the laboratory right-handed coordinate system is assumed to be perpendicular to the plane boundaries of the medium and directed outwards.

In the scalar approximation,

$$u \frac{\mathrm{d}\tilde{I}(\tau, u, \phi)}{\mathrm{d}\tau} = -\tilde{I}(\tau, u, \phi) + \frac{1}{C_{\mathrm{ext}}^{\mathrm{eff}}} \int_0^{2\pi} \mathrm{d}\phi' \int_{-1}^{+1} \mathrm{d}u' \langle Z_{11}(\Theta) \rangle_\xi \tilde{I}(\tau, u', \phi'), \tag{14}$$

$$u \frac{\mathrm{d}\tilde{I}(\tau, u, \phi)}{\mathrm{d}\tau} = -\tilde{I}(\tau, u, \phi) + \frac{\tilde{\omega}^{\mathrm{eff}}}{4\pi} \int_0^{2\pi} \mathrm{d}\phi' \int_{-1}^{+1} \mathrm{d}u' a^{\mathrm{eff}}(\Theta) \tilde{I}(\tau, u', \phi'), \tag{15}$$

where

$$\Theta = \arccos[uu' + (1 - u^2)^{1/2}(1 - u'^2)^{1/2} \cos(\phi - \phi')] \tag{16}$$

is the scattering angle, φ is the azimuth angle of the propagation direction,

$$\tilde{\omega}^{\mathrm{eff}} = \frac{C_{\mathrm{sca}}^{\mathrm{eff}}}{C_{\mathrm{ext}}^{\mathrm{eff}}} \tag{17}$$

is the "effective single-scattering albedo",

$$C_{\mathrm{sca}}^{\mathrm{eff}} = 2\pi \int_0^\pi \mathrm{d}\Theta \, \sin\Theta \langle Z_{11}(\Theta) \rangle_\xi \tag{18}$$

is the "effective scattering cross section", and

$$a^{\mathrm{eff}}(\Theta) = \frac{4\pi}{C_{\mathrm{sca}}^{\mathrm{eff}}} \langle Z_{11}(\Theta) \rangle_\xi \tag{19}$$

is the "effective phase function" normalized according to

$$\frac{1}{2} \int_0^\pi \mathrm{d}\Theta \, \sin\Theta \, a^{\mathrm{eff}}(\Theta) = 1. \tag{20}$$

Eq. (15) is mathematically equivalent to the standard scalar RTE [18,19]. However, $C_{\mathrm{ext}}^{\mathrm{eff}}$, $C_{\mathrm{sca}}^{\mathrm{eff}}$ and $\tilde{\omega}^{\mathrm{eff}}$ do not have the same physical meaning as the extinction and scattering cross sections and the single-scattering albedo in the case of a non-absorbing host medium [23].

4. Concluding remarks

The results of this paper demonstrate once again the power of the microphysical approach to radiative transfer developed in [20,21]. Indeed, the microphysical approach allows one to derive the RTE rather than to guess it. This yields unambiguous and definitive analytical expressions for the participating quantities and ends the lasting controversy caused by the use of different heuristic approaches.

The resulting RTE (3) is remarkably similar to that in the case of a non-absorbing host medium [20,21] and, in fact, has an intuitively obvious structure. It is straightforward to use the integral-equation counterparts of Eqs. (3) and (6) in order to demonstrate that the physical meaning of the elements of the Stokes column vectors $\tilde{\mathbf{I}}(\mathbf{r}, \hat{\mathbf{q}})$, $\tilde{\mathbf{I}}_{\mathrm{d}}(\mathbf{r}, \hat{\mathbf{q}})$, and $\mathbf{I}_{\mathrm{c}}(\mathbf{r})$ entering Eqs. (3), (5), and (6) is exactly the same as in the case of a non-absorbing host medium (see Section 8.12 of [21]).

Furthermore, the generalized RTE also remains applicable in the case of external illumination in the form of a parallel quasi-monochromatic beam of light.

The elements of the phase and extinction matrices entering the generalized RTEs (3), (5), and (6) are given by Eqs. (48)–(63) and (72)–(78) of [16] coupled with a straightforward ensemble averaging procedure. The requisite elements of the amplitude scattering matrix are found by solving explicitly the Maxwell equations (e.g., [25,26]). The solution of the RTE then yields all quantities necessary to evaluate the electromagnetic energy budget of the entire scattering volume V (or any part of it) or to describe the time-averaged response of a well-collimated polarization-sensitive detector of electromagnetic energy placed inside or outside V.

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